Mathematics and Mobile Learning

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Abstract

This paper argues for an approach to mobile learning that leverages students' informal digital practices as resources for designing mathematics classrooms activities. We briefly describe two exploratory designs along these lines, one featuring the use of photos taken by students outside class and the other centered on their recording and analyzing videos of motion. We then present a case study analysis of two students working through portions of these activities in class and discuss the potential of the approach as well as challenges associated with its implementation.

Introduction

Mobile devices, such as smart phones and media players, are increasingly powerful, convenient and ubiquitous in the lives of young people. These devices have transformed the ways that people communicate, seek information and work with data of various forms. Yet, in the classroom context, mobiles are often seen as a threat to the serious work of school. This is an understandable reaction, as many of the typical informal uses of mobiles – texting, game playing, and media consumption – seem incompatible with or disruptive to the goals of education. When mobiles are incorporated into classroom settings, they are often used in limited ways, such as to reproduce existing school functions, acting as calculators, calendars or textbooks.

We argue for a different vision of mobile devices in education, one that sees young people's informal digital practices as holding great potential to transform practices of the mathematics classroom. Mobile devices are highly flexible computing devices, but much of their use falls into four basic practices (White, Booker, Martin & Ching, 2012): (1) *capturing and collecting* information and experiences across a variety of settings, through photos, audio and video recordings, numerical and text entry; (2) *communicating and collaborating* with others via phone, text, email and social networks; (3) *consuming and critiquing* media including music, photos, videos, games and text documents; and (4) *constructing and creating* personal forms of representation and expression through edited photos and videos, sketches, podcasts, blogs and so forth. Although these informal digital practices are typically oriented toward out-of-school topics of interest, we believe they can be readily mapped on to mathematical, scientific and engineering practices highlighted in the Common Core math and Next Generation Science Standards. For example, the informal practice of *capturing and collecting* maps onto the STEM practice of collecting data, while *communicating and collaborating* maps onto scientific and mathematical discourse and argumentation.

What are the benefits of bringing informal digital practices into the classroom context? First, we believe that students' informal mobile practices are richer than the mobile practices typically allowed in schools. Given the profound effects on thinking and learning that mobile devices have had in work and community contexts, it is only reasonable to explore the potential of these devices for educational purposes. Failure to do so would risk a significant missed opportunity. Second, we take as axiomatic the basic constructivist tenet that students build new knowledge from their existing knowledge (e.g., National Research Council, 2000). This simple idea suggests that, whenever possible, we should invite and incorporate students' existing competencies into novel learning environments, and provide appropriate activities to bridge their current understandings with hoped-for disciplinary learning. In pursuing this commitment, we follow a thread of research and development in the learning science that spans everyday physics (Bryce & Macmillan, 2005), verbal language play (Lee, 2012) and videogames (Gee, 2007). We also draw upon a rich body of work in mathematics education that urges educators to make use of students' significant out-of-school mathematics understanding (e.g., Bonotto, 2005; Civil, 2002; Nasir, Hand, & Taylor, 2008; Schoenfeld, 1991).

In this paper, we share data from a classroom intervention study where we sought to create a social and technical infrastructure that accomplished two basic functions. First, it should help students to use their existing, informal digital practices to bring pieces of their everyday experience—in the form of photos and videos—into the classroom. Second, it should provide scaffolds for students and teachers to create rich mathematical interactions around these artifacts. We focus our analysis on the nature of the interactions that our designs helped to support and enable, and the ways that students took up (or failed to take up) opportunities to engage in shared mathematical thinking and reasoning.

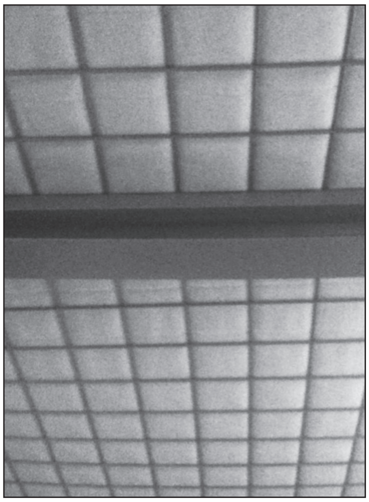
Capturing, Analyzing and Modeling Linear Phenomena with Photos and Video

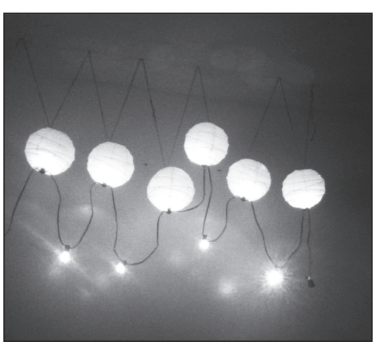
The study we describe below made extensive use of students' photos and videos. Student generated photographs have been used in science learning contexts to support student learning about health and nutrition (Reeve & Bell, 2009), as well as in mathematics classrooms as a support for student discussion and reflection on the nature of mathematics (Martin & Gourley-Delaney, 2013). Likewise, several design research projects have investigated the potential of videos to support students' mathematical modeling and analysis of motion, particularly in the context of technological resources for creating and manipulating mathematical representations of moving objects. Such technologies include motion detectors (Nemirovsky, Tierney & Wright, 1998), computer-based simulations (Roschelle, Kaput & Stroup, 2000) and digital video (Boyd & Rubin, 2006). In the case of digital video, Boyd & Rubin (1996) suggest that video-based tools provide a bridge between everyday perceptual experiences and mathematical representations of motion phenomena. They note that these distinct contexts differ in important ways with regard to the ways time, space, dimensionality and scale are experienced within each, and identify several key issues involved in learners' efforts to navigate between real-world phenomena and graphical representations using digital video tools. For example, they stress the utility of using real objects of known size in order to determine an appropriate scale for measuring distances in the video display. At the same time, they document the cognitive complexity learners face when integrating their knowledge about three-dimensional objects with their efforts to construct scales for interpreting the two-dimensional space of a video depiction. They also report that working with video of motion may predispose learners to using the horizontal axis of a corresponding graph to represent distance, a departure from the more standard practice in school math contexts of graphing distance on the vertical axis against time on the horizontal.

Method

Context and Participants

This paper presents data from a classroom-based design experiment conducted with a mixed-age cohort of sixteen 7th, 8th and 9th grade students at a small urban charter school with a diverse, predominantly low-income student population and an innovative, inquiry-based curriculum. The authors shared duties as the teachers in this study, taking turns leading the students through four days of instructional activities focused on linear functions and their graphs over a two-week period. Two student pairs in each class were selected as focus groups and videotaped during all activities. All screen states of the public computer display were recorded as a video file for each class session, and an additional camera with a wide zoom setting captured this projected display along with the whiteboard at the front of the room, as well as whole-class discussions and other teacher moves.





Figures 1, 2, and 3: Candidate Photos on Olivia's iPod

Tools and Learning Activities

Students participating in the unit were loaned an iPod touch for the duration of the two-week study. Beginning on the first day and continuing throughout the instructional sequence, the students were introduced to linear functions and graphs through classroom activities using *GroupGraph*, an iOS app version of the *Graphing in Groups* collaborative graphing environment for classroom networks developed by the first author (White, Wallace & Lai, 2012). Over the course of the unit, the students were given out-of-class homework assignments to use the built-in camera tools on the iPods first to capture photos of lines with a variety of slopes, and then to record videos of linearly varying phenomena. These pictures and videos then became materials that the students used in subsequent class sessions for further mathematical investigation. For example, the *GroupGraph* app includes a tool that allows students to upload photos taken on their iPods into a public graphing display, so one activity involved student pairs selecting one of their photos and trying to construct and determine the slope of one or more lines in the image. Another activity involved using iPads and a pair of commercially available apps, *Video Physics* and *Graphical*, to annotate the student-generated videos with time-sequenced Cartesian coordinate points that could then be exported into data tables and graphical displays.

Results

To illustrate the ways students engaged with these activities, we present three brief episodes from the video record of a pair of 8th grade students who worked together regularly throughout the study. The first episode, taken from the third day of the unit, highlights the students' work with photos of lines. The second episode captures a segment of student dialogue later in the same session as the class prepared to transition from photo to video work. The third episode focuses in on one of the two students' efforts to capture and analyze a video clip.

Episode One: Photos of Lines

At the end of the second class session, Tobin asked students to use their iPods to take photos of lines with different slopes as "homework" to be shared and investigated when they returned to class. Early in the third session, Lee asked each pair to use the *GroupGraph* app to select one of these photos to upload to the network for display in their group's graphing window. The following excerpt focuses on two students, Olivia and Jane, as they begin this task:

1. Olivia: Okay, so these are the photos that I have [shows her iPod to Jane]. I have um… that…[opens Figure 1 on her iPod]

2. Jane: That would probably be good.

3. Olivia: That…[swipes through photo album to display Figure 2]

4. Jane: Is this stuff in your room?

5. Olivia: And that [swipes again to show Figure 3]. No, it's my sister's room. So which one do you wanna use, the one with the [holds hand up and makes a pitched line] stripes?

6. Jane: Yeah. Then we have different [trace a crisscross with her index finger] lines to go to. [A cropped version of the cabinet door from Figure 2 appears in Jane and Olivia's graphing window in the public display (Figure 4)].

This brief exchange affords some insights into the ways students made sense of this photo-taking assignment. Olivia captured three images—one of a grid that might be a tile floor (Figure 1), one of a cabinet door (Figure 2), and one of a decorative string of globe lights hanging from wires at diagonals to one another (Figure 3). In each case, Olivia seems to have made a point of seeking out not just a single line to photograph, but multiple lines in parallel, perpendicular and/or skewed sets. In other words, she chose scenes that allowed her to fulfill the requirements of the assignment (take pictures of lines with different slopes) within a single image. She and Jane then further applied these same criteria to select the second image because it offered "different lines to go to" (line 6).

As students' photos began appearing in the public display, Jane and Olivia proceeded to construct a line that matched one of those on the cabinet door:

7. Jane: [begins moving her point up and down along the y-axis, from the origin]

8. Olivia: What do you think the great…? [Jane pauses her point at (0, 3), right at the intersection of two lines in the photo image. Olivia begins moving left, approaching one of those two lines] Let's do the biggest one right [moves left again to (-4, -2), so that her point is now directly over the same line as Jane's] there.

9. Jane: This one?

10. Olivia: Like that?

11. Jane: Mark it?

12. Olivia: You can go higher if you want. [Olivia moves her own point up one unit] You see—you see where I am? Go on that line. You should go probably up two [Jane moves her point up two units to 0, 5) and then across, like…I don't know…one, two,…

13. Jane: [moves to the left] Like that?

14. Olivia: No, across the other way.

15. Jane: [moves right to (1,4)]

16. Olivia: One more. Go up.

17. Jane: [moves over and up to (2, 5), then back down] Ah!

18. Olivia: It's okay. [Takes the iPod from Jane and moves her point to (3, 7), hovering just above and beyond the tip of the line on the cabinet door in a position that matches Olivia's own relative to the other end of the same line]. There we go. [returns the iPod, then picks it up again to mark on it and then on her own to form y=(8/7)x+3.57 (Figure 4)]

Beginning in line 7, we can observe how these students set about making mathematical sense of the images they collected. In particular, they were quick to identify one of the longest line segments in their selected image (line 8), and to use it as a frame of reference for forming their own *Graphing in Groups* line. Much of their subsequent interaction, as they proceeded to navigate the Cartesian grid superimposed on this image, involved reliance on what Stroup, Ares and Hurford (2005) describe as "electronic gestures." Electronic gestures are movements with button pushes or screen taps that overlap with deictic markers in participants' speech (as, for example, in Olivia's "right there" as she moves over the intended segment in line 8, and Jane's "like that" accompanying her move left in line 13) to contextualize and index conversation about a virtual space. Of course, these efforts at multimodal expression were not always sufficient to allow participants to successfully coordinate their actions as they collaborated in this novel environment, as we see in this episode when Olivia simply took Jane's iPod over to complete the final steps of forming their line herself (line 18).

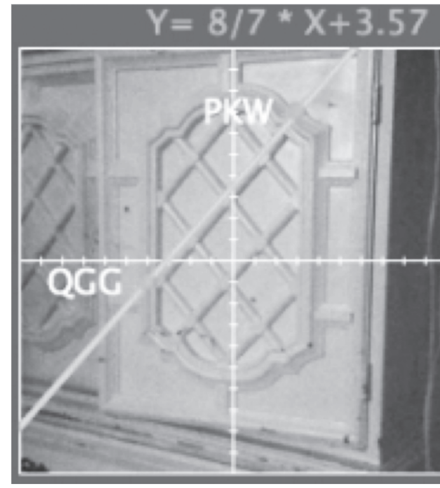


Figure 4: Olivia and Jane's Graphing Window with Cropped Photo Background

Episode 2: Transitioning from Photos to Video

Near the end of the photo activities, Lee took a moment to sum up the work the students had been engaged in to that point, and then prepared to transition to the next activity, focused on videos:

19. Lee: [Walks toward projector display on board] So here [points to the line displayed in Jane and Olivia's graphing window] we were working on mapping, matching lines to pictures [with palm flat and pitched to match the slope, moves his hand up and down two times along the displayed length of Jane and Olivia's line], where you literally can try to get the line drawn on top of the picture [repeats sweeping gesture over line three more times as he speaks, then walks several steps away from the display, facing class], and trying to think about how that relates to slopes—positive [sweeps right index finger up and right], negative [sweeps finger down and right], and flat [sweeps flat hand forward, palm down]. Now we're going to try it with video. So how many of you took a video on your iPod?

20. Olivia: [raises hand]

21. Lee: Great.

22. Olivia [to Jane]: You know what would be a really good video? An escalator.

23. Jane: [nods]

24. Olivia: 'Cause it's going [moves right hand in a smooth diagonal motion starting at the bottom and going up] …

25. Jane: Up.

26. Olivia: On a consistent … [repeats hand movement] it's on a slope and it's [repeats gesture again] going consistent.

27. Jane: I could take one of like a—one going down [snakes right hand downward], one going up [curves fingers back up].

28. Olivia: Uh huh.

In this short sequence, the teacher's summary comments about how they had been deriving lines from their photos—and his accompanying gestural depictions of lines with different slopes—appears to spark some reflections between Olivia and Jane about the kind of video they might collect. Indeed, both students appeared to be watching closely and listening intently, from their seats in the front row of the classroom, as Lee described the class's recent activity and highlighted the girls' own work in line 19. A moment later, Olivia appeared to take up and mirror Lee's gestural activity as she posed her idea for capturing video of an escalator (line 22), moving her flattened hand up along an imagined diagonal line three times as if at once enacting the escalator's motion and tracing the graph of a line representing that motion (lines 24, 26). Likewise, Jane's response with the suggestion that she could take videos of escalators going both "down" and "up" (line 27) featured a quick sequence of hand motions, synchronized with those alternating directional prepositions, that closely echoed Lee's gestural depiction of lines with both positive and negative slope.

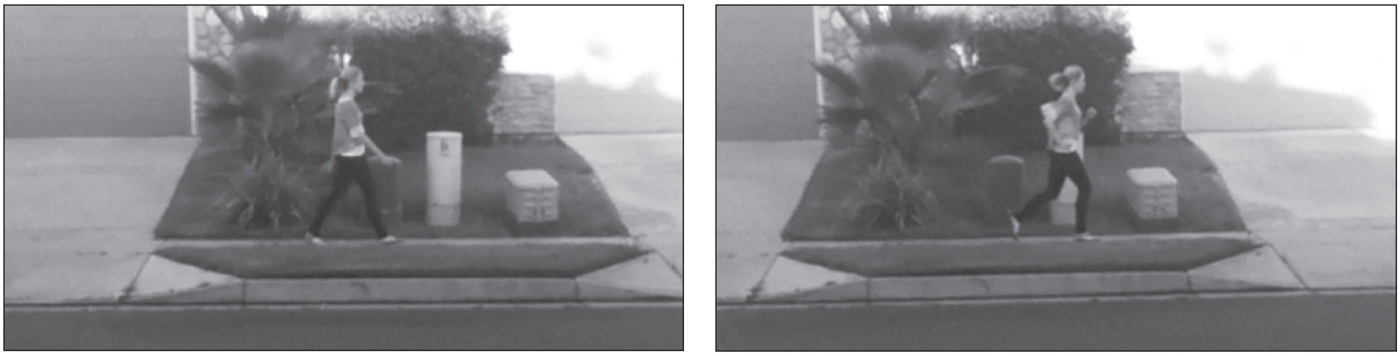
Even before the class had actually begun working with any videos collected by students, we can see some of the ways these students were reasoning about the criteria for "good" phenomena to capture varying linearly, and about the nature of that variation. In particular, Olivia pointed to both the uniformly upward slope of the motion and the "consistent" speed of the escalator's progression. While her gestural depiction of the escalator path indicates that she may have been primarily envisioning the way the sloped path of the escalator had the *shape* of a line—its height varying linearly over its length—the deliberately smooth and steady motion with which she traced that path and her description of its "consistent" progression suggest she may also have been intuitively drawn to the novelty of this exemplar as demonstrating vertical distance varying linearly with both horizontal distance *and* time.

It is also important to note the central roles played by gesture in this sequence: (a) in the teacher's comments, to generalize across the family of lines generated by students' various photographs and corresponding *Graphing in Groups* construction efforts; (b) in the students' comments, to illustrate motion phenomena in the absence of an actual video; and (c) in the interchange, where the students' uptake and paraphrase of the teacher's gestures serve as a bridge between linear photo and linear videos. One interpretation is that gestures helped the participants to explain the photos and videos. We note a second interpretation—that the introduction of photos and videos created an opportunity for speech and gesture about lines and slopes. In either case, the two mutually constitute the learning environment.

Episode 3: Capturing and Analyzing Videos of Linear Variation

On day three, the bulk of class time was spent on the photo activities; the session ended with Lee demonstrating how to use the *Video Physics* app to inscribe time-stamped points on a video for subsequent analysis using footage he captured with his own iPad. On day four, the students went to work applying this process to the videos they had captured on their own outside class. Olivia, as it turned out, did not record an escalator. Instead, she made two different short movies of a friend walking along a sidewalk in a residential neighborhood, shot from the opposite side of the street. In the first of these videos, the friend walked at a very steady rate throughout the 7-second clip (Figure 5a); Olivia began recording once her friend was already moving at this uniform speed, and held her camera steady and kept the walker at the vertical center of the screen as she moved from left until she disappeared at the right edge of the image. The second clip opened with an image of the same sidewalk. It was empty for several seconds until the friend appeared again on the left edge, initially walking at the same steady pace as in the previous video, but then accelerating to a jog by the middle of the screen, and a full sprint by the time she disappeared at the right edge (Figure 5b).

We can draw several inferences from these two short clips about Olivia's developing ideas about capturing linearly varying phenomena with video. Despite the apparent conflation of time and horizontal displacement as independent variables in her description of an escalator in the previous episode, here in the first video example, she appears to have identified a kind of phenomenon that would produce a linear graph of distance over time—uniform, unidirectional motion captured from a stationary camera. In the second example, she appears to have taken pains to hold the other elements of the film constant while introducing acceleration—an indication that she was actively exploring ideas about variation over time as she made her movies.



Figures 5a and b: Still images from Olivia's videos of a friend walking, and accelerating to a run.

In class, Olivia elected to focus on the first of these video clips, showing the walker moving at a uniform rate, to annotate using *Video Physics* and plot the resulting position versus time graph. The final excerpt below finds her in conversation with the first author, who had come by her desk to help her to set an appropriate scale for her plot using an object of known length in her video.

29. Tobin: You could use…

30. Olivia: Nineteen inches.

31. Tobin: Yeah, and get it on her stride, or you could just do her height. It might actually be easier to just do the height.

32. Olivia: Her height?

33. Tobin: Yeah. So…

34. Olivia: Ok. Why would her height be anything to like, you know.

35. Tobin: So you, you're going to go from, like her bottom, the bottom of her foot to the top of her head?

36. Olivia: Yeah, but is she walking sideways [Holds her right hand out, thumb down, index and middle fingers stretched forward as if to illustrate legs parallel to the ground, then sweeps to the left and right]?

As this segment began, Tobin began to offer a suggestion for a scale value (line 29), which Olivia anticipated by remembering the distance of 19 inches she had estimated as the length of her friend's stride when making the video the night before (line 30). Tobin, worried that she might find it difficult accurately mark this stride length on the video, proposed that she might instead consider using the friend's height as a known distance to apply to the image as a scale (line 31). Puzzled, Olivia asked why her height was relevant (lines 32, 34). Failing to recognize the source of her hesitation, Tobin began walking her through how to go about marking the height in the app (line 35). Olivia then posed her question in a different way, asking why the walker's height would be relevant to modeling her *horizontal* distance traveled over time unless she were "walking sideways" (line 36).

This clarifying moment in their conversation sheds some additional light on the challenges associated with interpreting the size of objects alongside motion and distance within the two-dimensional frame of the video display. Tobin (line 31) suggested, based on assumptions about the equal horizontal and vertical scales in this rectangular display (pixels are square), that known heights and lengths were equally relevant to setting the scale. Olivia, however, was clearly less certain about this equivalence, which suggests that even in the context of working with objects of known size when analyzing video of motion, issues of scale and dimensionality need considerable unpacking for learners. We also note that, as in the previous examples, gesture played an important role in illustrating Olivia's efforts to reason through this relationship between vertical orientation and horizontal motion—indeed, she managed to use her fingers to both enact that relationship, and to iconically capture what she saw as the absurdity of a person's "height" in horizontal motion (Figure 6). Given the established role of gesture in mathematical reasoning (Radford, 2003; 2009; Roth, 2001), the affordances of gesture for expressing ideas at the interface of motion and mathematical representation illustrated in these episodes, and the gestural nature of the user interface in the multi-touch displays of Tablet PCs, we posit that there might be considerable opportunities for designing tools and instructional activities that better leverage gestural resources to support student learning in this domain.

Figure 6. Olivia illustrates "walking sideways."

Discussion and Conclusion

Mobile devices have many features that make them exciting platforms for learning: their computing power, their affordability, their ubiquity and the affection they inspire in young people. We are particularly compelled by the suite of informal digital practices that has emerged around these devices. These practices, when coupled with the fact that students can carry these devices with them as they move across contexts, creates the potential to build bridges between the everyday objects of students' interest, and the mathematics of the classroom. In this paper, we describe and analyze our efforts to explore this promising space. Using devices, software and our roles as teachers, we created a social and technical infrastructure designed to enable students to leverage familiar mobile practice to capture, share and mathematize commonplace objects and events through photos and videos.

In our analysis, we examined a sequence of three episodes where a pair of students tried to make mathematical sense of linear phenomena. We looked not for the effectiveness of these tools in driving particular learning outcomes, but rather their efficacy in enabling interactions around mathematical ideas. We make two primary observations. First, we note the activities afforded students various, and sometimes challenging, opportunities to make sense of how mathematics could be superimposed onto the artifacts under consideration. For example, in the third segment, Olivia had to overcome her initial doubts that vertical length was relevant to the horizontal scaling task she had in front of her. Here, orientation was an important consideration, even when working with objects of known size; this extends the findings of Boyd and Rubin (1996).

Second, we found that gesture played a pivotal role throughout the interactions. This suggests that the suite of tools and representations provided both a communicative need to gesture, as well as a gestural canvas with which to work. Given the importance of touch and gesture for using mobile and other touchscreen devices, we see better knowledge of the mathematical uses of gesture in this space as a critical design resource.

Most important, we hope that this exploratory work serves as both a proof of concept for, and a call for further research on, the incorporation of students' informal mobile practices into the mathematics classroom.

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